Active $Q$ control in tuning-fork-based atomic force microscopy

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The authors present comprehensive theoretical analysis and experimental realization of active $Q$ control for the self-oscillating quartz tuning fork (TF). It is shown that the quality factor $Q$ can be increased (decreased) by adding the signal of any phase lag, with respect to the drive signal, in the range of $\theta_i$ to $\theta_i + \pi (\theta_i + \pi$ to $\theta_i + 2\pi$), where $\theta_i$ is the characteristic constant of TF. Experimentally, the nominal $Q$ value of $4.7 \times 10^3$ is decreased to $1.8 \times 10^3$ or increased to $5.0 \times 10^4$ in ambient condition, where the minimum detectable force is estimated to be $4.9 \times 10^{-14}$ N at 1 Hz. The novel $Q$ control scheme demonstrated in the widely used quartz TF is expected to contribute much to scanning probe microscopy of, in particular, soft and biological materials. © 2007 American Institute of Physics. [DOI: 10.1063/1.2753112]

The quartz tuning fork (TF) is widely used as a force sensor in scanning probe microscopy (SPM), such as near-field scanning optical microscopy, atomic force microscopy (AFM), electrostatic force microscopy, and magnetic force microscopy. In particular, for biological and soft materials, the TF sensor is regarded as an alternative imaging probe due to its high stiffness and discrimination property: The large spring constant is essential for controlling the tip-sample distance stably in the range of 1–10 nm. Moreover, in dynamic mode, the TF oscillates by an electrical drive without an additional actuator, which provides compactness and stability of the system.

However, when the TF is used in liquid as a shear-force sensor or as a qPlus sensor, the quality factor $Q$ as well as the detection sensitivity are significantly deteriorated. On the other hand, the $Q$ factor should be decreased for scanning in vacuum or at low temperature because high $Q$ results in slow response in feedback control. Therefore, it is very crucial to control the $Q$ factor properly, which is the main subject of the so-called $Q$ control. Despite its importance, the $Q$-control scheme has not been analytically treated in the TF-based AFM.

In this letter, we first make a general theoretical study by employing the TF’s equivalent circuit model in the dynamic-mode TF-based AFM, from which the comprehensive theory of $Q$ control is obtained. Our results show that the $Q$ value can be varied by adding a signal of proper phase lag with respect to the drive signal. In particular, we find that even a simple follower (inverter) (Ref. 2) without the external phase shifter is good enough to increase (decrease) the $Q$ factor. Then, experimental realization of such results is demonstrated by controlling the $Q$ factor as well as the decay time with respect to the gain and phase lag. The resultant enhanced sensitivity and the minimum detectable force are also estimated.

The TF device is generally analyzed by an electrical equivalent circuit which is consisted of a parallel circuit of a series $LRC$ and a stray capacitance $C_0$. Figure 1 shows the $Q$-control schematic diagram of the self-oscillating TF-based AFM with a self-excitation gain. We can write down the equation of motion as follows:

\begin{align}
L\dddot{I}_1 + R\ddot{I}_1 + \frac{I_1}{C} = \dddot{V} + GR_0(\dot{I}_1 + \dot{I}_2), \\
I_2 = C_0[\dot{\dddot{V}} + GR_0(\dot{I}_1 + \dot{I}_2)], \\
\dot{I} = I_1 + I_2,
\end{align}

where $G$ is a complex gain with the effects of the phase shifter included, $R_0$ is the resistance of the $I/V$ converter, and $L$, $R$, $C$, and $C_0$ are the intrinsic electrical properties of TF. With the substitution of $I = A\exp[i(\omega t + \phi)]$ and $V = V_0\exp[i(\omega t)]$, we obtain the general solution of amplitude $A$ and phase $\phi$ of $I$,

\begin{align}
Ae^{i\phi} = \frac{i\omega I_0/Q}{1 - i\bar{G}\bar{\omega}C_0} + \frac{1}{1 - \bar{\omega}^2 + i\bar{\omega}(\bar{Q}C_0\bar{\omega}^2\bar{G}/Q - i\bar{\omega}\bar{G}\bar{\omega}(1 - \bar{\omega}^2) + 1)},
\end{align}

where $\bar{\omega} = \omega/\omega_0$, $\omega_0 = 1/\sqrt{LC}$, $C_0 = C_0/C$, $\bar{G} = GR_0C_0\omega_0$, $Q = Lw_0/R$, and $I_0 = V_0/R$. By putting $\bar{G} = |\bar{G}|e^{i\phi}$ in Eq. (4) and comparing with the general Lorentzian solution of a driven damped oscillator, the effective quality factor ($Q'$) and resonance frequency ($\omega_0'$) can be approximated as, for small $C_0$ and $|\bar{G}|$,

\begin{align}
Q' \approx \frac{Q}{1 - |\bar{G}|(C_0\sin \theta + Q\cos \theta)},
\end{align}

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In Eq. (6), $|G_0| \sim 10^{-3}$ and $|G| \sim 10^{-4}$ for our setup, so that $\omega'_0$ is only slightly altered. However, the change of $Q'$ is quite noticeable. From Eq. (5), a map showing the qualitative change of $Q'$ can be obtained (Fig. 2). It shows that the $Q$ enhancement ($Q$ reduction) is achieved by adding any phase lag in the range of $\theta_1$ to $\theta_1 + \pi$ ($\theta_1 + \pi$ to $\theta_1 + 2 \pi$) with respect to the drive signal. This means that one can use the simple follower (inverter) (Ref. 12) even without using the additional phase shifter to increase (decrease) the $Q$ factor, which is the main advantage of the $Q$-control scheme of the TF-based AFM over the cantilever-based one, where the exact phase lag of $\pi/2$ has been implemented. Moreover, theoretically, if a suitable $|G|$ is obtained, $Q'$ can be enhanced or suppressed indefinitely.

Figure 3(a) presents the typical experimental resonance curves, where the enhanced $Q$ of 9090 and reduced $Q$ of 3333 are obtained at the phase lag of $17^\circ$ and $197^\circ$, respectively. Note that even if the phase shifter is set to zero phase lag (i.e., $\theta_0 = 0$), a residual phase difference of $17^\circ$, which is due to the electrical components in the gain part, is added to the output channel. In case of enhanced $Q$, we have observed that the resonance frequency shift is very small [Eq. (6)] and the resonance curve is well fitted to the symmetric Lorentzian. For decreased $Q$, however, the shift is not so negligible and the Lorentzian is not a good approximation to the asymmetric response except near the resonance. Note that if one intends to symmetrize the asymmetric line shape, additional electronics such as a bridge circuit may be employed.14

However, in order to achieve quantitatively the $Q$ enhancement or $Q$ reduction of TF, such a supplementary circuit is not required. Figure 3(b) plots experimentally measured $Q$ versus gain at the given intrinsic phase lags of $17^\circ$ (□) and $197^\circ$ (■), in good agreement with the corresponding theoretical results. Experimentally, we could increase $Q$ from the nominal value of $4.7 \times 10^3$ up to $5.0 \times 10^4$ and also decrease down to $1.8 \times 10^3$ in ambient condition, where $R_0 = 10 \pm \Omega$, $|G|=0.045$, and $\theta = 17^\circ$ for the $Q$-enhancement case, whereas $|G|=0.25$ and $\theta=197^\circ$ for the $Q$-reduction case. Here $|G| = |G|_{R_0 C_0} = |G|_{(R_0 I_0)/V_0} = |G| (0.413 \times 10^{-2})$, where $Q = 4600$, $V_0 = 4 \text{mV}_{\text{rms}}$, and $R_0 I_0 = 76 \text{mV}_{\text{rms}}$ ($I_0$ and $V_0$ are the driving amplitudes). Note that there is negligible difference between the theoretical results for $0^\circ$ and $17^\circ$ (or between $180^\circ$ and $197^\circ$), which indicates that the simple follower (inverter) can be employed even in the presence of its intrinsic undesirable small phase lag.

$Q$ enhancement (reduction) is a practical means not only to obtain the narrow (broad) resonances but also to increase (decrease) the decay time which determines the bandwidth of the amplitude measurement, as given by $\tau = 2Q/\omega_0$. Experimental results of the closely related temporal behavior of decay time with respect to the gain are presented in Fig. 4(a). As can be observed, the decay time is increased (decreased) with the increase (decrease) of $Q$, which agrees well with the theoretical results [Fig. 4(b)]. We also have found that there

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**FIG. 1.** $Q$-control schematics of the electrically driven tuning-fork-based atomic force microscopy. The total current $I$ is converted to a voltage by the $U/V$ converter and the phase shifted signal is added back to the drive signal after gain control, which realizes the self-excitation gain.

**FIG. 2.** (Color online) Phase map representing the effective $Q$ change for a given phase lag. The bright (dark) region indicates the phases where $Q$ is enhanced (reduced). Here $\tan \theta_0 = -Q/C_0$ and $\theta_0 = 81.12^\circ$ for a small conventional TF.

**FIG. 3.** (Color online) (a) Typical resonance curves where $Q$ can be measured. The solid curve represents the nominal case of no gain control ($|G|=0$). (b) Experimental results of $Q$ control versus gain at the phase difference of $17^\circ$ ( ), $Q$ enhancement) and $197^\circ$ ( ■, $Q$ reduction), in good agreement with the presented theoretical results.
is no appreciable difference between the theoretical results for 0° and 17° (or between 180° and 197°), as in the case of the $Q$ factor. Note that, by controlling the decay time, the scanning bandwidth can be easily adjusted, which allows the AFM imaging to be performed conveniently in various conditions with an enhanced or reduced $Q$.

Let us now discuss the sensitivity of TF in terms of the flat power spectrum $S_F$, which is given by

$$S_F^{1/2} = \left( \frac{2kk_BT}{\pi Q f_0} \right)^{1/2}.$$  

In our experiment, $S_F^{1/2} = 4.89 \times 10^{-14} \text{ N/Hz}$, where $k_B = 1.38 \times 10^{-23} \text{ J/K}$, $k = 1484 \text{ N/m}$, $T = 300 \text{ K}$, $Q = 5 \times 10^4$, and $f_0 = 52742 \text{ Hz}$. From this, the minimum detectable force within the bandwidth $B$ can be calculated as

$$F_{\text{min}} = S_F^{1/2} B^{1/2},$$

which becomes $4.9 \times 10^{-14} \text{ N}$ for 1 Hz bandwidth. In principle, with a suitable choice of $|G|$, the $Q$ factor can be enhanced greatly so that the minimum detectable force can be further decreased.

In conclusion, we have presented a quantitative analytical description, by employing the equivalent circuit model of TF, of the active response of the self-oscillating quartz-TF-based AFM, which implements a self-excitation gain. $Q$ enhancement ($Q$ reduction) is obtained at a phase lag in the range of $\theta_1$ to $\theta_1 + \pi$ ($\theta_1 + \pi$ to $\theta_1 + 2\pi$). In particular, it turns out that the simple follower (inverter), instead of the phase shifter, can be employed to increase (decrease) the $Q$ factor. With the active control of $Q$ from $1.8 \times 10^3$ to $5.0 \times 10^4$, the sensitivity of TF can be enhanced up to $4.9 \times 10^{-14} \text{ N/Hz}$ in ambient condition. Therefore, one can either perform a more sensitive force measurement by using the TF at a higher $Q$ value, or obtain the higher-quality scanning images at low temperature or in vacuum by using a lower-$Q$ TF. This quantitative and active $Q$-control scheme of the quartz TF is expected to be especially useful for various SPM studies of soft matter, biological sample, and water meniscus.

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12. The follower (inverter) is the active electrical component which produces a fixed phase lag of 0° (180°).
13. Equation (4) can be approximated to the Lorentzian form by ignoring the first term ($C_0$) and the $1-\omega^2$ term in the brackets. Comparing to the Lorentzian, $\omega_0$ is approximated to the angular frequency, which makes the real part of the denominator of Eq. (4) vanish, whereas $Q$ is given by the proportional coefficient of $1/\omega$ in the imaginary part.