열 및 통계 물리 2 (수시고사 3)

출제교수명: 정형채 시행일자: 1998. 3. . 요일

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1.[10pts.] Consider a system of two identical ptls., each of which can be in the $r = 1, 2$, or 3 states of respective energies, ϵ_1, ϵ_2, and ϵ_3. The system is in contact with a heat reservoir with $T = 1/\beta$.

(a)[3pts.] What is the partition function Z, if ptls obey BE statistics?

(b)[3pts.] What is the partition function Z, if ptls obey FD statistics?

(c)[4pts.] Calculate the mean number of ptls in the state 2 for the FD cases when $\epsilon_1 = 0, \epsilon_2 =$T, and $\epsilon_3 = 2T$.

2.[10pts.] Consider a Fermion gas of N identical ptls in contact with a heat reservoir with $T = 1/\beta$.

(a)[4pts.] Show that the mean number of ptls in the state s is given by

$$n_s = -\frac{1}{\beta}\frac{\partial}{\partial \epsilon_s}\log Z, \tag{1}$$

where $Z(N)$ is the partition function of the system.

(b)[6pts.] In the class, we showed that $Z(N)$ could be written as

$$\begin{aligned} Z(N) &= Z_s(N) + Z_s(N-1)e^{-\beta\epsilon_s} \\ &= Z_s(N)[1 + e^{-\beta(\epsilon_s - \mu)}], \end{aligned} \tag{2}$$

where μ is the chemical potential and Z_s is the partition function of a system idential to the given system except it does not have the state s. Find n_s in terms of β, ϵ_s and μ from Eqs. (1) and (2). How can you calculate μ?