Thermal and Statistical Physics 2 (Homework set 5)

Due: 3 p.m., 2012. 06. 11.

Collage: Department: Student Number: Name:

1. Consider an ideal gas of bosons whose density of states is given by $g(E) = CE^{\alpha-1}$ for some constants C and $\alpha > 1$. Derive an expression for the critical temperature T_c, below which the gas experiences Bose-Einstein condensation.

2. For classical ideal gas, average particle number $n_{\vec{k}}$ for an eigen state $|\vec{k}\rangle$ is given by

$$n_{\vec{k}} = e^{-\beta\left(\frac{\hbar^2 k^2}{2m} - \mu\right)}$$

(a) Express chemical potential μ and fugacity z in terms of density of particles, $n = N/V$.

(b) Grand partition function of classical ideal gas is given by

$$Z_G(T, V, \mu) = e^{\frac{V}{\lambda^3} e^{\beta\mu}}$$

Using, $N = \frac{\partial}{\partial(\beta\mu)} \ln Z_G$, find the fugacity $z = e^{\beta\mu}$ for classical ideal gas in terms of density $n = N/V$.

3. Assume that there are three eigenstates $\epsilon_1 = 0$, $\epsilon_2 = \epsilon$, and $\epsilon_3 = \frac{\ln 3}{\ln 2}\,\epsilon$, for a single particle. Answer the following questions when two fermions are in this system.

(a) Provide the occupation number representations for all possible states.

(b) They are in equilibrium with a heat reservoir with temperature $T = \frac{\epsilon}{\ln 2}$. Calculate partition function Z and find the average particle number n_l at ϵ_l state $(l = 1, 2, 3)$.

(c) They are in equilibrium with a particle and heat reservoir with temperature $T = \frac{\epsilon}{\ln 2}$ and fugacity $z = e^{\beta\mu}$. Show that the average particle number n_l^G in the state-l can be written as

$$n_l^G = \frac{1}{\frac{1}{z}\, b_l + 1}$$

and find b_l.

4. A system has two energy levels with energies 0 and ϵ. These can be occupied by (spinless) bosons from a particle and heat bath with temperature T and chemical potential μ. The bosons are non-interacting. Calculate the grand partition function Z_G in terms of z, T, and ϵ.

5. In Quantum Statistical thermodynamics, Grand Canonical Ensemble is most frequently used while Canonical Ensemble is frequently used in Classical case. Why do you think different ensembles are used ?