열 및 통계 물리 2 (기말 문제풀이)

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자연과학 대학 학과 학년, 학번: 성명:

1.

$$H = \sum_{i=1}^{N}\sum_{\alpha=1}^{3}\frac{p_{i,\alpha}^2}{2m} + \sum_{i=1}^{N}\sum_{\alpha=1}^{3}\frac{1}{2}\kappa\, x_{i,\alpha}^2.$$

(a)

$$\begin{aligned} Z_N &= \frac{1}{N!h^{3N}}\int\prod_{i=1}^{N}\prod_{\alpha=1}^{3} dp_{i,\alpha}dx_{i,\alpha}\exp(-\beta H) \\ &= \frac{1}{N!h^{3N}}\left[\int dp_i\, \exp(-\frac{\beta p_{i,\alpha}^2}{2m})\int dx_i\, \exp(-\frac{\beta\kappa\, x_{i,\alpha}^2}{2})\right]^{3N} \\ &= \frac{1}{N!h^{3N}}[2m\pi/\beta]^{3N/2}[2\pi/\beta\kappa]^{3N/2} \\ &= \frac{1}{N!}\left[\frac{2\pi T\sqrt{m/\kappa}}{h}\right]^{3N} \end{aligned}$$

(b)

$$\begin{aligned} F &= -T\,\log Z \\ &= -T\,[3N\log(2\pi T\sqrt{m/\kappa}/h) - N\log N + N] \\ \mu &= \frac{\partial F}{\partial N} \\ &= T\left[\log N\,-\,3\log\frac{2\pi T\sqrt{m/\kappa}}{h}\right] \end{aligned}$$

(c)

$$\begin{aligned} Q &= \sum_N \zeta^N Z_N \\ &= \sum_N\left[\zeta\left(\frac{2\pi T}{h}\sqrt{\frac{m}{\kappa}}\right)^3\right]^N\,/N! \\ &= \exp\left[\zeta\left(\frac{2\pi T}{h}\sqrt{\frac{m}{\kappa}}\right)^3\right] \\ \Rightarrow N &= \zeta\frac{\partial[\log Q]}{\partial\zeta} \\ &= \zeta\left(\frac{2\pi T}{h}\sqrt{\frac{m}{\kappa}}\right)^3 \\ \Rightarrow \zeta &= N/\left(\frac{2\pi T}{h}\sqrt{\frac{m}{\kappa}}\right)^3 \\ \mu &= T\log\zeta \\ &= T\left[\log N\,-\,3\log\frac{2\pi T\sqrt{m/\kappa}}{h}\right] \end{aligned}$$

2. (a)

$$\begin{aligned} E_1 &= \epsilon_1+\epsilon_2 = T\log 2 \\ E_2 &= \epsilon_1+\epsilon_3 = T\log 3 \\ E_3 &= \epsilon_2+\epsilon_3 = T\log 2 + T\log 3 = T\log 6 \\ Z &= e^{-\log 2}+e^{-\log 3}+e^{-\log 6} \\ &= \frac{1}{2}+\frac{1}{3}+\frac{1}{6} = 1 \end{aligned}$$

(b)

$$\begin{aligned} \bar{n}_1 &= [1\cdot e^{-\log 2}+1\cdot e^{-\log 3}+0\cdot e^{-\log 6}]/Z = \frac{1}{2}+\frac{1}{3} = \frac{5}{6} \\ \bar{n}_2 &= [1\cdot e^{-\log 2}+0\cdot e^{-\log 3}+1\cdot e^{-\log 6}]/Z = \frac{1}{2}+\frac{1}{6} = \frac{4}{6} \\ \bar{n}_3 &= [0\cdot e^{-\log 2}+1\cdot e^{-\log 3}+1\cdot e^{-\log 6}]/Z = \frac{1}{3}+\frac{1}{6} = \frac{3}{6} \end{aligned}$$

(c) $\sum_r n_r = N$으로 부터

$$\frac{1}{e^{-\beta\mu}+1}+\frac{1}{2e^{-\beta\mu}+1}+\frac{1}{3e^{-\beta\mu}+1} \;=\; 2$$

$\Rightarrow e^{-\beta\mu} = 27/100,$ 즉 $e^{\beta\mu} = 100/27.$

(d)

$$\begin{aligned} n_1 &= \frac{1}{e^{-\beta\mu}+1} = \frac{1}{\frac{27}{100}+1} = \frac{100}{127} \\ n_2 &= \frac{1}{2e^{-\beta\mu}+1} = \frac{1}{2\frac{27}{100}+1} = \frac{100}{154} \\ n_3 &= \frac{1}{3e^{-\beta\mu}+1} = \frac{1}{3\frac{27}{100}+1} = \frac{100}{181} \\ n_1/\bar{n}_1 &= 6\cdot 100/5\cdot 127 = 600/635 \approx 0.945 \\ n_2/\bar{n}_2 &= 6\cdot 100/4\cdot 154 = 600/616 \approx 0.974 \\ n_3/\bar{n}_3 &= 6\cdot 100/3\cdot 181 = 600/543 \approx 1.105 \end{aligned}$$

3.

(a)(b) 수시고사(3) 풀이 참조.

(c)

$$\begin{aligned} \epsilon_{F} &= \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3} \\ \Rightarrow n &= \frac{[2m\epsilon_{F}/\hbar^2]^{3/2}}{3\pi^2} \\ &= \frac{[2mc^2k_{B}T_{F}/(\hbar c)^2]^{3/2}}{3\pi^2} \\ &= \frac{[(2)(5\times 10^5 eV)(300/11605)eV/(1973eV\text{Å})^2]^{3/2}}{3\pi^2} \\ &\approx \frac{[(10^6)(1/400)/(4\times 10^6)]^{3/2}}{30}\text{Å}^{-3} \\ &= [2^4 10^2]^{-3/2}/30\text{Å}^{-3} = 1/[(2^6)(10^3)(3\times 10)]\text{Å}^{-3} \\ &\sim 10^{-7}\text{Å}^{-3} = 10^{17}cm^{-3} \end{aligned}$$

4. 생략