Effects of breaking-induced currents on refraction–diffraction of irregular waves over submerged shoal

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Abstract

To investigate the accuracy degeneration of conventional wave models for the case of breaking waves over a shoal, a system of numerical models is constructed by combining wave and flow models. The wave model employs the parabolic equation considering wave and current interaction. The flow model employs the shallow water equations including radiation stresses of waves. Numerical tests show that the breaking waves induce jet-like strong currents behind the shoal, and the currents, in turn, defocus the waves. As a result, the calculated wave height distribution agrees much closer to the measured data than that obtained using conventional wave models. Thus the effect of breaking-induced currents on the refraction–diffraction of waves over a shoal is clearly understood.

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Keywords: Irregular waves; Refraction–diffraction; Breaking-induced currents; Defocusing

1. Introduction

For accurate simulation of waves propagating over a shoal a numerical model, which can deal with refraction–diffraction of waves, is indispensable. Among various
wave models the parabolic equation model is widely employed due to its simple nature of numerical scheme and considerable accuracy of the solution. The parabolic equation, developed first by Radder (1979) for monochromatic waves, is continuously improved by a number of researchers to include nonlinear effect (Yue and Mei, 1980; Kirby and Dalrymple, 1983, 1986a; Liu and Tsay, 1984), bottom friction (Liu, 1986), wave breaking (Kirby and Dalrymple, 1986b), and wave–current interaction (Kirby, 1984). The range of validity of the parabolic equation for propagation direction is extended by Booij (1981) and Kirby (1986) to deal with the waves propagating with a large angle to the main wave propagation direction.

Recently, the parabolic equation is employed to simulate the irregular waves with wide-band frequency and directional spectra by virtue of linear superposition of a number of monochromatic component waves. Panchang et al. (1990) conducted comprehensive numerical simulations using a parabolic equation model for the experimental cases of irregular wave deformation over an elliptic shoal performed by Vincent and Briggs (1989) and found that the model gives reasonably accurate solutions for non-breaking waves.

Kirby and Özkan (1994) developed a parabolic wave model, REF/DIF S, and applied it to the Vincent and Briggs’ experimental case of breaking waves. Since the input wave height is large, a nonlinear dispersion relationship, i.e. amplitude dispersion, is implemented in their simulation. Their numerical model failed to predict the strong defocusing of waves behind the shoal which appears in the experimental data of Vincent and Briggs (1989). Kirby and Özkan believed that the discrepancy between the data and the model results arises because the breaking model used in REF/DIF S does not take directional effects into account.

Recently, Kirby et al. (1998) and Chen et al. (2000) developed a fully nonlinear Boussinesq wave model and applied the model to the breaking of monochromatic waves over a circular shoal. A strong jet-like current induced by breaking of waves was calculated. However, the effect of the current on the deformation of waves was not investigated.

Judging from the numerical experiences reported earlier, it is concluded that the accuracy of the conventional wave models degenerates significantly for breaking waves. In the present study, the reason for the accuracy degeneration of conventional wave models is analyzed and a mechanism involving breaking-induced current acting on the refraction–diffraction of waves over the shoal is proposed.

In the following section, a system of numerical models combining wave and flow models is introduced with a brief description of governing equations. In Section 3, the experiments of Vincent and Briggs (1989) are summarized. Then, in Section 4, the solution technique of the numerical model is presented. In Sections 5 and 6, some numerical simulations are conducted for the experimental cases of Vincent and Briggs (1989) to check the performance of the wave model, and the accuracy degeneration of conventional wave models in the cases of breaking waves is analyzed. In Section 7, numerical simulations are conducted including the effect of breaking-induced currents. Concluding remarks are made in the final section.
2. Governing equations

In the present study, the REF/DIF S model developed by Kirby and Özkan (1994) is used for the simulation of irregular waves. The model employs a wide-angle parabolic equation considering the Doppler shift due to current given as

\[
\begin{align*}
(C_{gn} + U)(A_n)_x - 2\Delta_1 V (A_n)_y + i(\bar{k}_n - a_0 k_n)(C_{gn} + U)A_n \\
+ \left\{ \frac{\sigma_n}{2} \left( \frac{C_{gn} + U}{\sigma_n} \right)_x - \Delta_1 \sigma_n \left( \frac{V}{\sigma_n} \right)_y \right\} A_n \\
+ i\Delta'_n \left[ ((CC_g)_n - V^2) \left( \frac{A_n}{\sigma_n} \right)_y \right] - i\Delta_1 \left\{ \left[ UV \left( \frac{A_n}{\sigma_n} \right)_y \right] + \left[ UV \left( \frac{A_n}{\sigma_n} \right)_x \right] \right\} \\
+ \frac{w_n}{2} A_n + \alpha A_n + \frac{-b_1}{k_n} \\
\times \left\{ \left[ ((CC_g)_n - V^2) \left( \frac{A_n}{\sigma_n} \right)_y \right] + 2i \left( \sigma_n V \left( \frac{A_n}{\sigma_n} \right)_x \right) \right\} \\
+ b_1 \beta_n \left\{ 2i\omega_n U \left( \frac{A_n}{\sigma_n} \right)_x + 2i\sigma_n V \left( \frac{A_n}{\sigma_n} \right)_x - 2UV \left( \frac{A_n}{\sigma_n} \right)_{xy} \right\} \\
+ \left[ ((CC_g)_n - V^2) \left( \frac{A_n}{\sigma_n} \right)_y \right] \\
- \frac{i}{k_n} \{ b_1 (\omega_n V)_y \} \\
+ 3(\omega_n U)_x \left( \frac{A_n}{\sigma_n} \right)_x - \Delta_2 \left\{ \omega_n U \left( \frac{A_n}{\sigma_n} \right)_x + \frac{1}{2} \omega_n U_x \left( \frac{A_n}{\sigma_n} \right) \right\} \\
+ i k \omega_n U (a_0 - 1) \left( \frac{A_n}{\sigma_n} \right) \\
= 0
\end{align*}
\]

where

\[
\beta_n = \frac{(k_n)_x}{k_n^2} + \frac{(k_n((CC_g)_n - U^2))_x}{2k_n^2((CC_g)_n - U^2)}
\]

\[
\Delta_1 = a_1 - b_1, \quad \Delta_2 = 1 + 2a_1 - 2b_1, \quad \Delta' = a_1 - b_1 \frac{k_n}{\bar{k}_n}
\]

In the above equations, the subscript \( n \) denotes the \( n \)th wave component. \( A_n \) is the complex amplitude, \( k_n \) the wave number, \( \bar{k}_n \) the reference wave number given along the wave incidence boundary, \( C \) the phase velocity, \( C_g \) the group velocity, \( w_n \) the dissipation function for bottom friction, and \( a_0, a_1 \) and \( b_1 \) the directional correction factors given by

\[
a_0 = 1, \quad a_1 = -0.75, \quad b_1 = -0.25
\]

\( \sigma_n \) is the intrinsic frequency to take into account the Doppler effect due to currents.
and is determined by the following dispersion relationship:

\[ \sigma_n^2 = (\omega_n - k_n U)^2 = g k_n \tanh k_n h \]  

(5)

where \( \omega_n \) is the absolute frequency, \( g \) the gravitational acceleration, and \( h \) the local water depth. \( \alpha \) is the energy dissipation coefficient for wave breaking given by Thornton and Guza (1983) as

\[ \alpha = \frac{3\sqrt{\pi}}{4} \frac{\bar{f} B^3}{\gamma^4 h^5} H_{\text{rms}}^5 \]  

(6)

where \( \bar{f} \) is a representative frequency and \( H_{\text{rms}} \) a root-mean-squared (rms) wave height. \( B \) and \( \gamma \) are the empirical coefficients and are chosen to be 1 and 0.6, respectively, as recommended by Chawla et al. (1998). When the currents are neglected, (1) is reduced to the parabolic equation developed by Kirby (1986).

After the wave field is solved, the radiation stresses can be evaluated by

\[ S_{xx} = \frac{1}{2} \rho g \sum_{n=1}^{N} |A_n|^2 \left\{ \frac{C_{gn}}{C_n} (1 + \cos^2 \theta_n) - \frac{1}{2} \right\} \]  

(7)

\[ S_{yy} = \frac{1}{2} \rho g \sum_{n=1}^{N} |A_n|^2 \left\{ \frac{C_{gn}}{C_n} (1 + \sin^2 \theta_n) - \frac{1}{2} \right\} \]  

(8)

\[ S_{xy} = \frac{1}{4} \rho g \sum_{n=1}^{N} |A_n|^2 \frac{C_{gn}}{C_n} \sin 2\theta_n \]  

(9)

where \( N \) represents the total number of wave components, and \( \theta_n \) is the angle of wave propagation. Using the radiation stresses the current field can be calculated by the following modified shallow water equations:

\[ \frac{\partial \tilde{\eta}}{\partial t} + \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = 0 \]  

(10)

\[ \frac{\partial P}{\partial t} + \frac{\partial (P^2/D)}{\partial x} + \frac{\partial (PQ/D)}{\partial y} + gD \frac{\partial \tilde{\eta}}{\partial x} + \frac{1}{\rho} \left( \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} \right) \]  

\[ + \frac{g n^2}{D^{7/3}} \sqrt{P^2 + Q^2} = \nu_t \left( \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} \right) \]  

(11)

\[ \frac{\partial Q}{\partial t} + \frac{\partial (PQ/D)}{\partial x} + \frac{\partial (Q^2/D)}{\partial y} + gD \frac{\partial \tilde{\eta}}{\partial y} + \frac{1}{\rho} \left( \frac{\partial S_{xy}}{\partial x} + \frac{\partial S_{yy}}{\partial y} \right) \]  

\[ + \frac{g n^2}{D^{7/3}} \sqrt{P^2 + Q^2} = \nu_t \left( \frac{\partial^2 Q}{\partial x^2} + \frac{\partial^2 Q}{\partial y^2} \right) \]  

(12)

where \( \tilde{\eta} \) represents the mean free surface displacement measured upwards from still water level. \( P \) and \( Q \) are the volume fluxes in horizontal dimensions. \( D(= h + \tilde{\eta}) \) is
the total water depth, \( n \) is the Manning’s roughness coefficient, and \( v_e \) is the eddy viscosity. The horizontal velocity components, \( U \) and \( V \), can be obtained using

\[
U = \frac{P}{D}, \quad V = \frac{Q}{D}
\]

(13)


Vincent and Briggs (1989) conducted comprehensive experiments on the deformation of regular and irregular waves over an elliptic shoal. Waves are generated using the directional spectral wave generator (DSWG) in Coastal Engineering Research Center of US Army Corps of Engineers. Fig. 1 shows the experimental setup of Vincent and Briggs (1989).

For the generation of irregular waves, the following directional spectrum is used:

\[
E(f, \theta) = S(f)D(\theta)
\]

(14)

where \( f \) and \( \theta \) denote, respectively, the frequency and propagation direction of a component wave. \( S(f) \) is the TMA shallow-water frequency spectrum (Bouws et al., 1985) and \( D(\theta) \) is the wrapped normal spreading function (Borgman, 1984). More details on the experimental set-up and the directional spectrum for wave generation can be found in Vincent and Briggs (1989).

The experiments of Vincent and Briggs are conducted for three kinds of incident waves with different directional spreading, i.e. unidirectional (U—series), narrow (N—series) and broad (B—series) directional spreading. For each case of directional spreading, both the bandwidth of frequency and the nonlinearity of incident

Fig. 1. Location of elliptic shoal and wave height measurement sections (Vincent and Briggs, 1989).
waves are varied. The measured wave heights for each case are compared with those of monochromatic (M—series) waves, which can serve as a reference to show the effects of frequency and directional spreading and the nonlinearity of incident waves on the wave deformation due to submerged shoal.

4. Numerical scheme

The REF/DIF S model employs the Crank–Nicolson scheme to get the finite difference representation of (1). The computational domain (i.e. \(0 \leq x \leq 19\ m\), \(0 \leq y \leq 25\ m\) with the center of shoal located at \(x = 6.1\ m\) and \(y = 12.5\ m\)) is discretized by a finite difference grid with \(\Delta x = \Delta y = 0.05\ m\). The range of frequency spreading is divided into 10 frequency components with different value of \(\Delta f\) for each component to have equal wave energy. The range of directional spreading is divided into 20 directional components. As a result, the irregular waves are disintegrated with a total number of 200 component waves. Each wave component is assumed to be monochromatic with a single frequency and an angle of incidence. Each wave component is calculated independently. However, when the energy dissipation rate is calculated, all the components are superimposed to get root-mean square wave heights. The interested reader is referred to Kirby and Özkan (1994) for more details on the computational scheme implemented in REF/DIF S model.

For the current field (10)–(12) are solved with a finite difference leap-frog scheme on a staggered grid system (Goto and Shuto, 1983). The same grid size for wave field is used. The Manning’s coefficient \(n\) and the eddy viscosity \(v_t\) are chosen to be equal to 0.014 m/s\(^{1/3}\) and 0.006 m\(^2\)/s, respectively. The Manning’s coefficient chosen in this study is reasonable because the bottom of the wave basin is finished with mortar. However, different values are also tested. The time step of 0.01 s is determined by Courant stability condition. Since the current field is symmetric about the centerline of the shoal, only the half domain is solved. The last half of the current field can be obtained by mirror image. As a result, the effect of jet meandering on the wave deformation is ignored.

The interactive procedure proposed by Haas et al. (1998) is employed to solve the combined modeling system and is repeated here. Firstly, the wave field is calculated using (1) with neglecting the current field, and the radiation stresses are evaluated using (7)–(9). Then, the current field is obtained using (10)–(12). The current field is, in turn, supplied for (1) to get a new wave field. Iterative computations can be made as a transient problem until a steady state is obtained. Strictly speaking, this procedure can be applied only to the steady state problem, because (1) is invalid for a transient wave field.

The breaking-induced current becomes strong as the time elapses, and the wave field is influenced by the current. Thus both wave and current fields should be calculated at each time step. However, computational experience shows that the evolution of current field is slow. To save computational time the wave field is calculated once every 500 time steps of the current field computation.
5. Test of wave model with non-breaking waves

The experimental input wave conditions for non-breaking series are listed in Table 1 (Vincent and Briggs, 1989). We remark here that the parameters, \( x \) and \( \gamma \), in Tables 1 and 2 are irrelevant to the values employed in (6). Those were used to control the wave height and frequency spectral peakedness of TMA spectra. Numerical simulations for the cases of irregular waves (i.e. U, N and B series) given in Table 1 are conducted to test the performance of the REF/DIF S model employed in the present study. In the numerical simulations, the wave-induced currents are neglected, and the nonlinear dispersion relationship of Stokes-to-Hedge type implemented in the REF/DIF S model is considered.

Figs. 2–4 show the comparisons of the calculated and measured significant wave heights normalized by incident wave height along section 4. The agreements between calculated and measured wave heights are fairly good. Thus the performance of the present model is promising for the simulation of non-breaking waves with various directional spreading.

6. Test of wave model with breaking waves

The parameters for the incident waves in the cases of breaking waves are listed in Table 2 (Vincent and Briggs, 1989). Fig. 5 presents the measured wave height distribution along section 4 for each case of input wave conditions (M3, N5 and B5). In the figure, the wave heights were normalized using the wave height measured at the gage 10 shown in Fig. 1. The measured wave heights at the gage 10 are presented in Table 2 (Briggs, 1987). The wave heights near the center of the section are observed to be small in comparison with those off the center.

To be consistent with the other cases presented by Figs. 2–4 the measured wave heights for each case of input wave conditions M3, N5 and B5 are normalized by the wave height generated by wave maker and are plotted in Fig. 6. Thus all the

Table 1
Input wave conditions for non-breaking waves (Vincent and Briggs, 1989)

<table>
<thead>
<tr>
<th>Input</th>
<th>Case ID</th>
<th>Peak period (s)</th>
<th>Significant wave height (cm)</th>
<th>( x )</th>
<th>( \gamma )</th>
<th>( \sigma_m (\text{cm}) )</th>
<th>Freq. spreading</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monochromatic</td>
<td>M2</td>
<td>1.3</td>
<td>2.54</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>Mono</td>
</tr>
<tr>
<td>Uni-directional</td>
<td>U3</td>
<td>1.3</td>
<td>2.54</td>
<td>0.00155</td>
<td>2</td>
<td>0</td>
<td>Broad</td>
</tr>
<tr>
<td></td>
<td>U4</td>
<td>1.3</td>
<td>2.54</td>
<td>0.00047</td>
<td>20</td>
<td>0</td>
<td>Narrow</td>
</tr>
<tr>
<td>Narrow-directional</td>
<td>N3</td>
<td>1.3</td>
<td>2.54</td>
<td>0.00155</td>
<td>2</td>
<td>10</td>
<td>Broad</td>
</tr>
<tr>
<td></td>
<td>N4</td>
<td>1.3</td>
<td>2.54</td>
<td>0.00047</td>
<td>20</td>
<td>10</td>
<td>Narrow</td>
</tr>
<tr>
<td>Broad-directional</td>
<td>B3</td>
<td>1.3</td>
<td>2.54</td>
<td>0.00155</td>
<td>2</td>
<td>30</td>
<td>Broad</td>
</tr>
<tr>
<td></td>
<td>B4</td>
<td>1.3</td>
<td>2.54</td>
<td>0.00047</td>
<td>20</td>
<td>30</td>
<td>Narrow</td>
</tr>
</tbody>
</table>
comparisons between calculated and measured data will be made using the data presented in Fig. 6.

Numerical computations are performed with and without breaking effects for two cases of irregular waves (i.e. N5 and B5 cases), and the resulting wave heights are compared with the measured ones in Figs. 7 and 8. In these computations the breaking-induced currents are intentionally neglected as in the usual case of conventional simulations. When the effect of breaking is excluded for the case of N5, the calculated wave heights behind the central part of the shoal are much larger than the measured one as shown in Fig. 7. The inclusion of breaking effect in the computation with $\gamma$ value of 0.6 as recommended by Chawla et al. (1998) does not improve the accuracy of the computation. The over-estimation of wave height near

<table>
<thead>
<tr>
<th>Input</th>
<th>Case ID</th>
<th>Peak period (s)</th>
<th>Wave height (cm) (wave maker)</th>
<th>$\alpha$</th>
<th>$\gamma$</th>
<th>$\sigma_m$ (°)</th>
<th>Wave height (cm) (gage 10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monochromatic</td>
<td>M3</td>
<td>1.3</td>
<td>13.5</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>12.12$^a$</td>
</tr>
<tr>
<td>Narrow-directional</td>
<td>N5</td>
<td>1.3</td>
<td>19.0</td>
<td>0.0262</td>
<td>20</td>
<td>10</td>
<td>15.85$^a$</td>
</tr>
<tr>
<td>Broad-directional</td>
<td>B5</td>
<td>1.3</td>
<td>19.0</td>
<td>0.0865</td>
<td>2</td>
<td>30</td>
<td>16.08$^a$</td>
</tr>
</tbody>
</table>

$^a$ Data from Briggs (1987).

![Fig. 2. Comparison of calculated and measured wave height distributions along section 4 for cases U3 and U4.](image)
the central part can be attributed by the lack of energy dissipation due to breaking on the top of the shoal. Thus the strength of energy dissipation increased by reducing the $\gamma$ value up to 0.4 in the breaking model. However, the over-estimation of
wave height is persistent as shown in Fig. 7. Moreover, the overall pattern of wave height distribution is totally different from the measured one. The same trend is obtained for the case of B5 as shown in Fig. 8. Thus it is found that the accuracy of the numerical model degenerates significantly for the cases of breaking waves.

Fig. 5. Measured wave height distributions along section 4 for each case of input wave conditions M3, N5 and B5 normalized by wave height measured at gage 10 (Vincent and Briggs, 1989).

Fig. 6. Measured wave height distributions along section 4 for each case of input wave conditions M3, N5 and B5 normalized using wave height generated by wave maker.
The accuracy degeneration of numerical model for breaking waves can be analyzed as in the followings: as shown in Fig. 9(a), the refraction pattern of waves around the shoal can be conceptually described by tracing wave rays. The wave energy entering the central part of the shoal is dissipated by wave breaking at the top of the shoal, while the wave rays entering both shoulders of the shoal are

Fig. 7. Comparison of calculated and measured wave height distributions along section 4 for case N5; breaking-induced current is excluded.

Fig. 8. Comparison of calculated and measured wave height distributions along section 4 for case B5; breaking-induced current is excluded.
refracted to form caustics in the rear side of the shoal. Thus the waves lose their energy first in the breaking zone, but due to converging wave rays the waves gain the energy to produce large wave heights at focusing zone where the transect 4 locates. However, this focusing of wave energy behind the shoal is non-physical phenomenon, which does not appear in a physical model.

Kirby and Özkan (1994) pointed out that the discrepancy between the numerical and the measured data is attributed to the inaccuracy of breaking model employed in their model. Since the breaking model of Thornton and Guza (1983) implemented in the REF/DIF S was developed based on the experimental data obtained in a one-dimensional wave flume, it suffers from the lack of the sense of direction. This idea of directional effect on the wave breaking is partially supported by the experimental observations conducted by She et al. (1994) and Nepf et al. (1998). They found that the onset and the severity of wave breaking are affected by the directional spreading of waves. To the best of authors’ knowledge the directional effect on wave breaking is not established yet to be included in the numerical model. Even though the directional effect is included in the breaking criteria of the REF/DIF model system, only the slight change of energy dissipation due to wave breaking is expected, and the change of overall pattern of wave climate behind the shoal will not be significant as shown in Figs. 7 and 8.

On the other hand Haas et al. (1998) performed a numerical simulation for the generation of rip currents on a barred beach. Haas et al. employed a combined numerical modeling system, REF/DIF model for short waves and SHORECIRC model (Van Dongeren et al., 1994; Van Dongeren and Svendsen, 1997) for currents. An iterative feedback between wave and current models is carried out to deal with the wave–current interaction problem. Haas et al. showed that significant changes in rip currents occur when the interaction is included in the modeling system because the wave field is influenced by the rip currents.

In the present study, inspired by the idea of Haas et al., the effect of breaking-induced current on the deformation of waves over the shoal will be included in the simulation. A mechanism involved in the situation of combined refraction and breaking of waves can be proposed using the refraction diagram depicted in Fig. 9(b). The breaking of waves over the shoal induces strong currents in the direction of wave propagation, and this breaking-induced currents, in turn, defo-

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**Fig. 9.** Schematic representation of wave breaking and refraction pattern over a shoal: (a) formation of numerical focusing zone; (b) formation of physical geometric shadow zone.
cuses the converging wave rays outwards from the central part behind the shoal. If there were no breaking-induced current, the converging wave rays cause the wave height to grow again in the focusing zone. However, instead of the focusing zone a geometric shadow zone appears there due to defocused wave rays, and two caustics are formed next to the shadow zone. As a result, it is possible to have a wave height distribution pattern along section 4 consistent with the measured data of Vincent and Briggs (1989). This mechanism is physically sound and more plausible to explain the discrepancy between the measured and calculated wave climates behind the shoal after the breaking of waves.

7. Numerical simulation considering breaking-induced currents

To check the validity of the present concept, numerical simulations are conducted for the same cases with considering the breaking-induced current in the computations. All calculations are carried out for the time span of 260 s during which the measured data are collected (Vincent and Briggs, 1989). The $\gamma$ value of 0.6 is employed for the breaking model of Thornton and Guza (1983) as used in REF/DIF S model with no adjustment. The Manning’s coefficient $n$ is set to be 0.014. Fig. 10 compares the calculated wave height distribution with measured data along section 4 for N5 case. When the breaking-induced current is included, a reasonable agreement between calculated and measured wave height distributions is observed, while the calculated data with neglecting the current field show a reversed distribution of wave height. However, a quantitative agreement is not achieved yet. The reason for this discrepancy can be explained by the fact that the breaking-induced current is stronger and narrower than the physical one. The Manning’s coefficient

![Fig. 10. Comparison of calculated and measured wave height distributions at 260 s along section 4 for case N5; breaking-induced current is considered ($\gamma = 0.6$).]
and the eddy viscosity can be tuned to achieve accuracy improvement of the numerical model. In general, the bottom friction and the eddy viscosity have a similar effect on the spreading of jet-like currents in such a way that the increase in bottom friction or turbulent intensity makes the jet spread more. As a result, the centerline velocity of the jet becomes weaker. Thus, in the present study, the combined effect will be tested as a whole by adjusting only the bottom friction for the currents. It is found by trial and error that the Manning’s coefficient of 0.028 gives a best agreement between calculated and measured data as shown in Fig. 10. The increase in bottom friction can be justified by the fact that the bottom friction under the wave–current interaction could increase significantly for both waves and currents. Thus the parameters of $\gamma = 0.6$ for breaking criterion and $n = 0.028$ for bottom friction will be consistently used for the subsequent computations.

Fig. 11 shows the evolution of calculated wave heights along section 4. At the initial stage, the breaking-induced current is absent and the calculated wave height distribution is totally different from the measured one. As time elapses the current becomes strong. As a result the wave heights in the central part are reduced, while those of both side parts are increased. Fig. 12 presents the evolution of wave height along the centerline of the shoal (i.e. sections 9 and 7 of Fig. 1). It is worthwhile to note here that the wave field reaches a steady state first near the top of the shoal where the breaking of waves is the strongest. The region of steady state develops slowly in the direction of wave propagation.

Fig. 13 represents the distribution of longitudinal velocity component of breaking-induced currents along section 4 for the case of N5. Fig. 14 shows the flow pattern of breaking-induced currents at $t = 260$ s. A strong jet along the centerline of the shoal is clearly visible.
Figs. 15 and 16 show the average angle distribution for the cases of excluding and including breaking-induced currents, respectively, in the computations. In these figures, the wave height and the predominant direction of energy flux are given by the magnitude and the direction of each arrow, respectively. When the currents are neglected, as shown in Fig. 15, the waves are breaking on the top of the shoal, and the focusing of wave energy after the wave breaking is developed behind the shoal. On the other hand, when the breaking-induced currents are included, the convergence of wave energy is effectively reduced or disappeared due

Fig. 12. Evolution of calculated wave height along sections 9 and 7 for case N5 ($\gamma = 0.6, n = 0.028$).

Fig. 13. Distribution of longitudinal velocity component of breaking-induced currents along section 4 for case N5.
to Doppler shift exerted by breaking-induced current behind the shoal as shown in Fig. 16. Thus the wave height along the centerline of the shoal is kept lower than those of other regions. These results support clearly the new concept represented by Fig. 9.

Fig. 17 presents the comparison of the calculated wave height distribution with measured data along section 4 for B5 case. The same parameters (i.e. $\gamma = 0.6$ for breaking criterion and $n = 0.028$ for bottom friction) as used for the case of N5 are
also employed in this computation. When the breaking-induced currents are included, the accuracy improvement of the numerical model is remarkably good. Fig. 18 shows the flow pattern of breaking-induced currents for B5 case.

Through the numerical computations for these two cases of breaking waves the effect of breaking-induced current on the refraction–diffraction of waves over the shoal is clearly shown. Judging from the numerical results with breaking waves, it is concluded that the breaking-induced currents should be considered in the

![Fig. 16. Average angle distribution for N5 case; breaking-induced currents is considered.](image)

![Fig. 17. Comparison of calculated and measured wave height distributions at 260 s along section 4 for case B5; breaking-induced current is considered (γ = 0.6, n = 0.028).](image)
numerical model for the accurate simulation of breaking waves over a shoal. Though one of the mechanisms controlling the deformation of breaking waves is found, further researches are indispensable to achieve accuracy improvements of the model. Topics for future research would include: the refinement of the breaking model through the tuning of empirical parameters and the implementation of directional effects in the breaking model, and some improvement of viscosity coefficients in the shallow water equations to take into account the effect of turbulence generated by wave breaking. These items are important to obtain an accurate shape of jet-like currents induced by breaking waves.

8. Conclusions and future research

A system of numerical models combining wave and flow models is constructed to investigate the accuracy degeneration of conventional wave models for the case of wave breaking over a shoal. The REF/DIF S model developed by Kirby and Özkan (1994) is employed as wave model which solves the parabolic approximation equation considering wave and current interaction. The flow model employs the shallow water equations including radiation stresses of waves. Numerical tests to the experimental cases of Vincent and Briggs (1989) show that the REF/DIF S model is sufficiently accurate for non-breaking waves. However, the accuracy of the model degenerates significantly for breaking waves. The reason for the accuracy degeneration of conventional wave models is investigated. When the interaction of wave and current is considered in the simulation, numerical results show that the breaking waves induce jet-like strong currents behind the shoal, and the currents, in turn, defocus the waves. As a result, the overall pattern
of wave height distribution agrees much closer to the measured data than that obtained using conventional models. Thus the effect of breaking-induced currents on the refraction–diffraction of waves over a shoal is clearly understood. A comprehensive three-dimensional physical experiment will be launched to support further the new concept presented in this study.

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