Technical Note

Water surface resonance in the L-shaped channel of seawater exchange breakwater

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Received 2 December 2002; accepted 16 April 2003

Abstract

The resonance period of the L-shaped channel in the caisson is predicted analytically for the seawater exchange breakwater of “Applicability Study of the Seawater Exchange Breakwater (1). Korea Ministry of Maritime Affairs and Fisheries (in Korean) (1999a)”. Hydraulic experiments are conducted for a composite breakwater with a rear reservoir that is one of the seawater exchange breakwaters developed by them. For regular waves, the water surface elevation in the channel and the flow rate through the breakwater are measured. For irregular waves, the flow rate through the breakwater and the reflection coefficient on the breakwater are measured. The resonant maximum values in both the surface elevation and the flow rate, and the resonant minimum values in the reflection coefficient are all at wave periods slightly longer than analytically predicted ones. The measured resonance period for irregular waves is closer to the predicted one than for regular waves. If the resonance period of the L-shaped channel is fitted to the dominant period of incident waves, there would be high efficiency of seawater exchange between inside and outside the harbor.

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Keywords: Seawater exchange breakwater; Resonance; Hydrodynamic prediction; Hydraulic experiment; Regular and irregular waves
1. Introduction

The breakwaters are to protect facilities in the harbor from storms and also to keep calm waters inside the harbor in order to moor ships and carry out the cargo-working conveniently. However, when pollutants from the land flow into the harbor in which the tidal effects are minor, there occurs low rate of seawater exchange between inside and outside the harbor resulting in environmental problems such as bad smell and ecological disorders. A thorough way to solve such problems is to shut off the source of the pollutants and to dredge a deposit of them. Another solution is to construct so-called seawater exchange breakwaters through which clean seawater with fresh air is allowed to flow into the harbor. Such breakwaters have been constructed since the 1970s in Japan. Multi-slit caisson breakwaters were installed in the Muroran and Nemuro Ports in Japan. Waters outside the harbor are allowed to flow through the slit of the caisson. This type of breakwater was also installed in the Pohang Port in Korea. Also, dual cylindrical caisson breakwaters were installed in the Nagashima Port in Japan. A donut-shaped wave chamber is formed between the outer permeable cylinder and the inner impermeable cylinder. Semi-circular caissons with circular holes were also installed in the Miyazaki Port. There are two types of flow system in the seawater exchange breakwater. One is to induce both-directional flow between inside and outside the harbor. The other one is to allow only one-directional flow from outside to inside the harbor. It is known that the one-directional flow system is better than the both-directional flow system in view of seawater exchange.

Recently, Lee et al. (1994) developed a circular channel breakwater to reduce wave overtopping and expedite seawater exchange between inside and outside the harbor. The circular channel in the caisson opens to outside the harbor and it is connected to a flow conduit, so that clean seawaters flow into the harbor. This breakwater has lower rates of both the wave reflection and overtopping compared to the conventional vertical breakwater. This breakwater would improve the water quality inside the harbor particularly in normal wave climate, if the resonance period of the channel is fitted to that of usually incoming waves. However, this breakwater is difficult to construct due to the circular geometry of the channel. Also, the channel must be long enough to get the resonance period equal to that of incoming waves. Thus, the caisson must be larger than the conventional one, which makes it more expensive.

Recently, Lee et al. (1999a) developed a seawater exchange breakwater which has L-shaped channels instead of circular channels in the caisson. The cell of the caisson can be used directly as the L-shaped channel that makes it easier to construct the breakwater than the circular channel breakwater. A reservoir in the caisson is located in front of or behind the channel to increase the flow rate through the breakwater. Lee et al. (1999b) compared structural stability of the seawater exchange breakwater to that of the conventional vertical breakwater by measuring wave forces and moments acting on the structure. Comparisons showed better stability of the new breakwater than the conventional one, in the case that the weights of the two breakwaters are equal to each other.
In this study, firstly, the resonance frequency of the L-shaped channel is predicted using the continuity equation and the integrated Euler equation along the streamline. Secondly, hydraulic experiments with both regular and irregular waves are conducted to see that, at the predicted resonance frequency, the water surface elevations in the channel blow up and thus we get higher rates of flow through the breakwater and lower reflection coefficients of waves on the breakwater. And finally, the present study is summarized briefly.

2. Resonance frequency of L-shaped channel

The resonance frequency of the L-shaped channel can be predicted by using the continuity equation and the integrated Euler equation along the streamline of the channel. Fig. 1 shows the configuration of the caisson with the channel.

Integration of the Euler equation along the streamline of the channel from the opening to the water surface yields the following equation

\[
\frac{0-p_1}{\rho} + g(\eta-z_i) + \frac{v_2^2-v_1^2}{2} + \int \frac{\partial v}{\partial t} \, ds = 0
\]  

(1)

where \( p \) is the pressure, \( \eta \) is the vertical elevation of water surface in the channel, \( v \) is the velocity along the streamline, \( \rho \) is the density of water, \( g \) is the gravitational acceleration, \( z \) is the vertical coordinate upward from the mean water level, \( t \) is the time, \( s \) is the coordinate along the streamline, and the subscripts 1 and 2 denote the opening and the water surface in the channel, respectively. Application of the continuity equation between the opening and the water surface gives the following equation

\[
Q = Av = A_1v_1 = A_2v_2
\]  

(2)

where \( Q \) is the flow rate and \( A \) is the cross-sectional area of the channel. Application of the linearized kinematic free surface boundary condition to the water surface gives the following equation

\[
v_2 = \frac{\partial \eta}{\partial t}
\]  

(3)

Fig. 1. Configuration of seawater exchange breakwater for obtaining resonance period of the channel.
Using Eqs. (2) and (3), the last term in the left side of Eq. (1) becomes
\[ \int \frac{\partial v}{\partial t} \, ds = \int \frac{\partial}{\partial t} \left( \frac{\partial \eta A_2}{\partial A} \right) \, ds = \frac{\partial^2 \eta}{\partial t^2} \int \frac{A_2}{A} \, ds \] (4)

Since \( A_1 = A_2 \), the continuity Eq. (2) yields that the two nonlinear terms of the kinetic energy in Eq. (1) may be canceled out by each other resulting in the following equation
\[ p_1 = \rho g (\eta - z_1) + \rho \frac{\partial^2 \eta}{\partial t^2} \int \frac{A_2}{A} \, ds \] (5)

When waves are approaching the breakwater with the angular frequency of \( \omega \), the pressure \( p_1 \) and the surface elevation \( \eta \) can be written as
\[ p_1 = |p_1| \cos(\omega t + \epsilon_p), \quad \eta = |\eta| \cos(\omega t + \epsilon_\eta) \] (6)

where \( \epsilon_p \) and \( \epsilon_\eta \) are the phase angles of the pressure \( p_1 \) and the surface elevation \( \eta \), respectively. Substitution of Eq. (6) into Eq. (5) gives the following equation
\[ |\eta| \cos(\omega t + \epsilon_\eta) = \frac{1}{1 - \omega^2 \int \frac{A_2}{A} \, ds} \left[ z_1 + \frac{|p_1|}{\rho g} \cos(\omega t + \epsilon_p) \right] \] (7)

If the denominator of the right side of Eq. (7) is zero, the magnitude of the surface elevation would become infinitely large irrespective of the pressure magnitude. This phenomenon is called the resonance of the channel. The resonance angular frequency \( \omega_r \) is given by
\[ \omega_r = \sqrt{\frac{g}{\int \frac{A_2}{A} \, ds}} \] (8)

And, the corresponding resonance period \( T_r \) is given by
\[ T_r = 2\pi \sqrt{\frac{\int \frac{A_2}{A} \, ds}{g}} \] (9)

If the cross-sectional area is uniform, i.e. \( A = A_1 = A_2 \), and the length of the channel is \( l \), then the resonance period would be given by
\[ T_r = 2\pi \sqrt{\frac{l}{g}} \] (10)

The resonance period given by Eq. (10) is the same as that of the oscillating liquid in a U tube with the length \( 2l \) of liquid column (Streeter and Wylie, 1979).
Using the Laplace equation and the free surface boundary condition, Lee et al. (1994) derived the resonance period of a circular channel given by

\[
T_r = 2\pi \sqrt{\frac{\left(\frac{\theta_i + \theta_o}{2} + \frac{\pi}{2}\right) r_i + r_o}{g \cos \left(\frac{\theta_i + \theta_o}{2}\right)}}
\]

where \( r \) is the radius of the channel wall, \( \theta \) is the angle between the horizontal line crossing the curvature center of the circular channel and the mean water level at the wall, and the subscripts i and o denote the inner and outer walls, respectively. If the curvature center of the channel is around the mean water level, then the following relations can be obtained as

\[
\frac{\theta_i + \theta_o}{2} \approx 0, \quad \left(\frac{\theta_i + \theta_o}{2}\right) + \frac{\pi}{2} \frac{r_i + r_o}{2} \approx l
\]

Therefore, the resonance period given by Eq. (11) is almost the same as that given by Eq. (10). This means that Eq. (9) can be applied to any shape of the channel.

Eq. (10) implies that the resonance period is proportional to the square root of the channel length. The equation also implies that the resonance period would be varied with the cross-sectional area of the channel. If the cross-sectional area increases from the opening, we would have the relation of \( \int_1^2 \left(\frac{A}{A_2}\right) ds > l \) and the resonance period would become longer than that given by Eq. (10). On the other hand, if the cross-sectional area decreases from the opening, the resonance period would become shorter than that given by Eq. (10).

3. Hydraulic experiment for the seawater exchange breakwater with a rear reservoir

Lee et al. (1999a) have developed various types of seawater exchange breakwater in addition to the circular channel breakwater. Experiments are conducted for the seawater exchange breakwater with a rear reservoir in the caisson. The reservoir that is located behind the channel is connected to the flow conduit that, in turn, is connected to the caisson wall inside the harbor. Fig. 2 shows the front view of the breakwater for experiment. The width of the caisson is 49 cm and the diameter of the flow conduit is 6 cm.

3.1. Equipment and conditions for hydraulic experiment

Fig. 3 shows the configuration of the wave flume with the breakwater and the measuring equipment. The wave flume is 53.15 m long, 1.25 m high, and 1.0 m wide. The mean water depth is 32 cm. A piston type wave maker has the maximum
moving distances of ±50 cm. The water quantity induced by the waves overtopping the breakwater will be returned to the up-wave of the structure through the pipes that are located under the breakwater. Fig. 3(c) shows an enlarged view of the breakwater and the measuring equipment. The measuring equipment of flow rate consists of a weir box, a box to store the water from the weir, and a pump to send the water in the storing box to an outside box. Up-wave of the weir box, a pipe is connected to the flow conduit in the caisson. And a check valve is located in the middle of the pipe to allow water to flow at only needed time interval. The incoming waves are both regular and irregular. Table 1 shows the conditions of regular waves in which measurements are made of the surface elevations in the channel and the flow rates through the breakwater. The circle sign means that experiments are conducted and the cross sign means experiments are not conducted. In order to measure both the flow rate through the breakwater and the reflection coefficient of waves on the
Table 1
Conditions of regular incident waves

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breakwater, irregular waves are made by the Bretschneider–Mitsuyasu spectrum of which the density is given by

$$S(f) = 0.257H_s^2T_s(fT_s)^{-5}\exp[-1.03(T_s^2)^{-4}]$$ (13)

where $H_s$ is the significant wave height, $T_s$ is the significant wave period, and $f$ is the frequency. In the present experiment, the conditions with $H_s = 4$ cm and $T_s = 1.0, 1.1, 1.15, 1.2, 1.4, 1.6, 2.0$ s are used.

3.2. Water surface elevation in the channel

After cutting off the flow from the channel to the reservoir, measurements are made of water surface elevations in the channel. The distance from the opening to the mean water surface is $l = 26.45$ cm. Eq. (10) gives the resonance period of the channel $T_r = 1.03$ s.

The maximum water surface elevations are normalized by the incident wave height. Fig. 4 shows the variation of the maximum normalized ones with the wave period. The variation with the incident wave height is also shown in the figure. The highest value of the maximum normalized surface elevation, i.e. resonance, occurs at periods between 1.1 and 1.4 s. The averaged resonance period is $T_r = 1.22$ s, that is 18% larger than the predicted one of 1.03 s. This difference may be because the flow is not smooth through the L-shaped channel and thus turbulent vortices occur in the channel. Therefore, the travel time of the flow in the channel becomes longer than the predicted one. In the case of a circular channel, the measured resonance period is almost equal to the predicted one (Lee et al., 1994). This is because the circular channel allows a smooth flow. When waves are reflected fully, the maximum normalized water surface elevation would be unity at which the dashed line is marked in the figure. The maximum normalized one is above unity for all the cases except
very short wave period. Especially, it is above 2 near the resonance period. This proves that the water surface elevation in the channel blows up near the resonance period.

3.3. Flow rate through the breakwater

Firstly, the flow rates through the breakwater are measured for regular waves. Variations are made with both the wave period $T$ and wave height $H$. Variations are also made with the height of the reservoir wall adjacent to the channel as $d = 2, 4, \text{ and } 6 \text{ cm}$. Fig. 5 shows flow rates through the breakwater for $H = 2.8 \text{ cm}$ and $H = 12 \text{ cm}$.
The wave period and the reservoir wall height are varied in this figure. For all the cases considered in this study, the maximum flow rate occurs at the resonance periods between 1.0 and 1.6 s. The averaged resonance period is $T_r = 1.34$ s, that is 30% larger than the predicted one of $T_r = 1.03$ s. The resonance period in flow rate is also larger than the resonance period of 1.22 s in water surface elevation. The fact that the measured resonance period in flow rate is much larger than the predicted one may have more reason in addition to the turbulent vortices expressed in Section 3.2. The cross-sectional area of the channel increases near the reservoir wall yielding the relation of $\int_{1}^{2} (A_2/A)ds > l$ which, in turn, gives a resonance period larger than that predicted by Eq. (10).

The results in Fig. 5 imply that, in the case of small wave height, the flow rate is larger with a lower reservoir wall than with higher one. On the contrary, in the case of large wave height, the flow rate is larger with a higher reservoir wall than with a lower one. If the wave height is small, waters in the channel cannot flow into the reservoir over a higher reservoir wall. The flow into the reservoir is allowed only over a lower wall. However, if the wave height is large, waters in the channel may flow into the reservoir over any sized wall and the reservoir with a higher wall may keep more waters thus giving larger flow rate.

Fig. 6 shows the variation of resonance periods with the wave height. The variation is also shown with reservoir wall height. As the wave height is larger, the resonance period becomes longer. If the wave height is larger, larger quantity of waters flow into the channel and the turbulent vortices may extend the travel time much more.

Secondly, the flow rates through the breakwater are measured for irregular waves. The significant wave height and the reservoir wall height are fixed as $H_s = 4$ cm.

![Fig. 6. Period of maximum flow rate through the breakwater versus height of regular waves; solid line with filled circle: $d = 2$ cm, dashed line with rectangle: $d = 4$ cm, dash-dotted line with plus: $d = 6$ cm.](image-url)
and $d = 2$ cm, respectively. The significant wave period is varied as $T_s = 1.0, 1.1, 1.15, 1.2, 1.4, 1.6, 2.0$ s. Measurements are made of the mean flow rate through the breakwater for 4 min.

Fig. 7 shows the variation of the flow rates with the significant wave period. The maximum flow rate is $Q = 354$ cm$^3$/s at the significant wave period of $T_s = 1.2$ s which is 17\% larger than the predicted resonance period of $T_r = 1.03$ s. It is interesting that the resonance period of $T_s = 1.2$ s for irregular waves is closer to the predicted one of 1.03 s than the resonance period of $T = 1.34$ s for regular waves. The maximum flow rate would be converted to 7.2 cm$^3$/s/cm per unit width.

If the model structure is to be reduced with the scale of 1/25 compared to that in the field, then the resonance period of $T_s = 1.2$ s would be 6 s in the field. Also, in the field, the caisson width would be 12.2 m, the pipe diameter would be 1.5 m, and the flow rate through each caisson would be 325 m$^3$/h/m. In reality, the significant wave periods between 6 and 9 s are common and the flow rate of 325 m$^3$/h/m would lead to seawater exchange between inside and outside the harbor significantly.

### 3.4. Reflection of waves on the breakwater

If there is a low degree of wave reflection on the breakwater, it is convenient to moor ships or carry out cargo-working in front of the breakwater. Measurements are made of the reflection coefficient of irregular waves on the seawater exchange breakwater. The significant wave height and the reservoir wall height are fixed as $H_s = 4$ cm and $d = 2$ cm, respectively. The significant wave period is varied as $T_s = 1.0, 1.1, 1.15, 1.2, 1.4, 1.6, 2.0$ s.

In order to get the reflection coefficient of waves, Park et al.’s (1992) method is used, which uses least squares method in separating the incident and reflected waves from the time series data of water surface elevations. They suggest using three wave

![Fig. 7. Flow rate through the breakwater versus significant wave period of irregular waves with $H_s = 4$ cm and $d = 2$ cm.](image-url)
gauges instead of two in order to get the solutions of the reflection coefficient in the whole frequency range. Three wave gauges with distances of 30 and 50 cm between the neighboring ones are installed between the wave maker and the breakwater. The fast Fourier transform method is used to get the spectral density of incident and reflected waves. Also, the smoothing techniques in Otnes and Enochson (1978) are used for the spectral analysis of the data. The 8192 data points are processed in 15 segments of 1024 points per segment. These segments overlap by 50% for smoother and statistically more significant spectral estimates. The raw spectra are then ensemble-averaged. Further smoothing is made by band-averaging over five neighboring frequency bands. The time step of the time series data is $\Delta t = 1/40$ s.

Fig. 8 shows the spectral densities of incident and reflected waves versus the wave frequency. Fig. 9 shows the corresponding reflection coefficients versus the component wave period. The cases with $T_s = 1.0, 1.2, 2.0$ s are shown in these figures. For all the cases with seven different significant wave periods, the reflection coefficients are minimal around the measured resonance period of $T_s = 1.2$ s.

Fig. 10 shows the weight-averaged reflection coefficients of irregular waves. In the figure, variations of the averaged values are shown with the input significant wave periods. On the whole, the reflection coefficients are lower than those with the

![Fig. 8. Spectral densities of incident and reflected irregular waves versus wave frequency; solid line: incident wave, dashed line: reflected wave.](image)
Fig. 9. Reflection coefficient of irregular waves on the breakwater versus component wave period.

Fig. 10. Weight-averaged reflection coefficients of irregular waves on the breakwater versus significant wave period.
conventional caisson breakwater. Especially, the minimal reflection coefficient is 0.55 at the resonance period of $T_s = 1.2$ s.

4. Conclusions

For the seawater exchange breakwater with the L-shaped channel, the resonance period of the channel is predicted using the continuity and integrated Euler equations along the streamline. Hydraulic experiments are conducted for both regular and irregular waves incident on the seawater exchange breakwater with a rear reservoir. For regular waves, the water surface elevation in the channel and the flow rate through the breakwater are measured. The measured resonance periods in surface elevation and flow rate are 18% and 30%, respectively, larger than the predicted one. The increase of the resonance period in terms of the surface elevation is reasoned that the flow in the channel is not smooth and the resulting turbulent vortices extend the travel time. The increase of the resonance period in terms of the flow rate is reasoned that, in addition to the turbulent vortices, the increase of cross-sectional area of the channel near the reservoir wall extends the travel time more.

For irregular waves, the flow rate through the breakwater and the reflection coefficient of waves on the breakwater are measured. The resonant maximum values in the flow rate and the resonant minimum values in the reflection coefficient are all at wave periods 17% larger than the predicted one. The measured resonance period for irregular waves is closer to the predicted one than for regular waves. For the case of small wave height, the reservoir with a lower wall has larger flow rate. On the contrary, for the case of large wave height, the reservoir with a higher wall has smaller flow rate. As the wave height is larger, the measured resonance period becomes longer. In that waves with larger height have longer periods in nature, the resonance periods of the breakwater in the field may be fitted to the periods of incident irregular waves in broad period range.

If the resonance period of the L-shaped channel is fitted to the dominant period of incident waves, there would be high efficiency of seawater exchange between inside and outside the harbor.

Acknowledgements

The first author wishes to acknowledge the financial support of the Korea Science and Engineering Foundation (No. R01-2000-000-00365-0). The second author wishes to acknowledge the financial support of the Korea Ministry of Maritime Affairs and Fisheries with the research title of “Applicability study of the seawater exchange breakwater”.
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